

In a Cartesian coordinate plane, which of the following straight lines is parallel to the line passing through the points (1, 0) and (0, 1)?

$$\underline{ax + by = c}$$

- ☐ A.  $2x + 3y = 0$
- ☐ B.  $x = 2$
- ☒ C.  $x + y = 3$
- ☐ D.  $x = y - 1$
- ☐ E.  $y = 1$

$$y = 3 - x$$

$$m = \frac{1 - 0}{0 - 1} = -\frac{1}{1}$$

$$m = -1$$

The polynomial  $12a^2 - 18b^2$  is exactly divisible by:

$$2a^2 - 3b^2$$

$$a\sqrt{2} - b\sqrt{3}$$

- ☐ A.  $a - b$
- ☐ B.  $12a + 18b$
- ☐ C.  $12a - 18b$

☒ D.  $\sqrt{2}a - \sqrt{3}b$  = 0

☐ E.  $\sqrt{6}(a - b)$

$$\rightarrow 6(2a^2 - 3b^2) = 0$$

$$\rightarrow \sqrt{2}a - \sqrt{3}b$$

Consider two distinct polynomial  $p(x)$  and  $q(x)$  of degree 3 with real coefficients. Which of the following statements always holds true?

$$p(x)^3 + p(x)^3 = (p+q)^3$$

$$p(x)^3 \cdot q(x)^3 = (q \cdot p)^3$$

- ☒ A. The degree of  $p(x) + q(x)$  is 3 and the degree of  $p(x) \cdot q(x)$  is 3
- ☐ B. The degree of  $p(x) + q(x)$  is 6 and the degree of  $p(x) \cdot q(x)$  is  $\leq 9$
- ☐ C. The degree of  $p(x) + q(x)$  is  $\leq 3$  and the degree of  $p(x) \cdot q(x)$  is 6
- ☐ D. The degree of  $p(x) + q(x)$  is 6 and the degree of  $p(x) \cdot q(x)$  is  $\leq 6$
- ☐ E. The degree of  $p(x) + q(x)$  is 3 and the degree of  $p(x) \cdot q(x)$  is  $\leq 6$

Given a real number  $x$ , the expression  $\frac{2^x \cdot 2}{\sqrt{4^{x+1}}}$  is equivalent to:

$$\frac{2^x \cdot 2}{\sqrt{4^{x+1}}} \rightarrow 2^{x+1}$$

$$\frac{2^{x+1}}{2^{x+1}}$$

$$\sqrt{4^{x+1}}$$

$$2 \sqrt{4^{x+1}} > 0$$

$$\frac{x+1}{2}$$

$$4^{\frac{x+1}{2}} \rightarrow 2^x$$

$$= 2^{x+1}$$

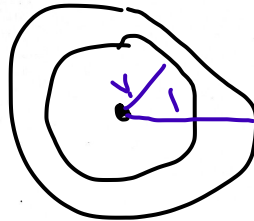
- ☒ A. 1  
☐ B. 0  
☐ C.  $\frac{1}{2^x}$   
☐ D.  $\frac{1}{2}$   
☐ E. 2



Two concentric metallic spheres have radii 1 and  $r$  (with  $r < 1$ ). Find the value of  $r$  such that the volume of the space between the smaller and the larger sphere is half of the volume of the larger sphere.

$$\frac{4}{3} \pi r^3 = \frac{4}{6} \pi$$

- ☒ A.  $r = \frac{1}{\sqrt[3]{2}}$
- ☐ B.  $r = \frac{1}{\sqrt{2}}$
- ☐ C.  $r = \frac{1}{\sqrt[3]{3}}$
- ☐ D.  $r = \frac{1}{\sqrt{3}}$
- ☐ E.  $r = \frac{1}{2}$



$$\frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi 1^3$$

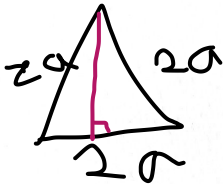
$$V_2 = \frac{4}{6} \pi$$

Consider an equilateral triangle  $A$  of side  $a$  and another equilateral triangle  $B$  of side  $2a$ . The area of  $B$  is:

$$S_B = \frac{\sqrt{3} a^2}{2}$$

$$h = \frac{\sqrt{3}a}{2}$$

$$S = \frac{\sqrt{3}a^2}{4}$$



$$h_2 = \frac{2\sqrt{3}a^2}{2} = \sqrt{3}a^2$$

- ☒ A. the double of the area of  $A$
- ☐ B. more than four times the area of  $A$
- ☐ C. greater than the area of  $A$ , but less than the double of the area of  $A$
- ☐ D. independent of the area of  $A$
- ☐ E. four times the area of  $A$

$$\frac{S_A}{S_B} = 2$$

Let  $0 \leq x \leq \frac{\pi}{2}$ . Then the solution of the equation  $\sqrt{3}\sin^2 x + \sqrt{3}\cos^2 x - 2\sin x = 0$  is:

جایگزینی ✓

$$0 \leq x \leq \frac{\pi}{2}$$

- ☐ A.  $x = 0$
- ☐ B.  $x = \frac{\pi}{6}$
- ☐ C.  $x = \frac{\pi}{4}$
- ☒ D.  $x = \frac{\pi}{3}$
- ☐ E.  $x = \frac{\pi}{2}$

$$\frac{\sqrt{3}}{4} + \frac{3\sqrt{3}}{4} - \frac{4}{4} = 0$$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \frac{2\sqrt{2}}{2} = 0$$

$$\sqrt{3} + 6 - 2 = 0$$

If  $f(x) = x^2 - x^3$  then  $f(x-2)$  is equal to:

$$(3-x)(x^2-4x+4)$$

$$f(x-2) = (x-2)^2 - (x-2)^3 = (x-2)^1$$

$$x^2 - 4x + 4 - (x^3 - 6x^2 + 12x - 8)$$

- ☒ A.  $(3-x)(x-2)^2$   
☐ B.  $x^2 - x^3 + 2$   
☐ C.  $x^2 - x^3 - 2$   
☐ D.  $x^2 - 2 - x^3 + 2$   
☐ E. none of the other answers

The inequality  $\sqrt[3]{x^3 + 8} < 0$  holds:

$$\sqrt[3]{x^3 + 2^3} \rightarrow x + 2 < 0$$

$$x < -2$$

- ☒ A. if and only if  $x < -2$
- ☐ B. if and only if  $x < 0$
- ☐ C. if and only if  $x < -1$
- ☐ D. if and only if  $x < 1$
- ☐ E. for no real values of  $x$

Sort the following numbers in ascending order:  $7$ ,  $\sqrt{47}$ ,  $\sqrt{3} + \sqrt{27}$

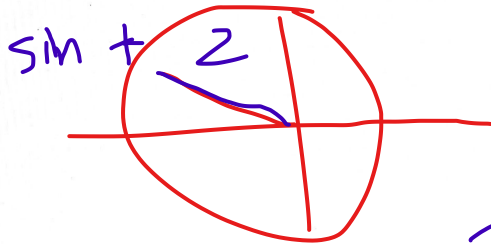
$$7 \quad \sqrt{47} \\ 6.85$$

$$\frac{\sqrt{3} + \sqrt{27}}{4\sqrt{3}} \\ 6.92$$

- ☐ A.  $\sqrt{47} < 7 < \sqrt{3} + \sqrt{27}$   
☐ B.  $7 < \sqrt{47} < \sqrt{3} + \sqrt{27}$   
☐ C.  $7 < \sqrt{3} + \sqrt{27} < \sqrt{47}$   
☐ D.  $\sqrt{3} + \sqrt{27} < \sqrt{47} < 7$   
☒ E.  $\sqrt{47} < \sqrt{3} + \sqrt{27} < 7$

An angle measures 2 radians. Therefore:

- ☒ A. the sine of the angle is positive ✓
- ☐ B. the angle is acute ✗
- ☐ C. the sine and the cosine of the angle have the same sign ✗
- ☐ D. the cosine of the angle is positive ✗
- ☐ E. the tangent of the angle is not defined ✗



$$2 \text{ R} = 1/4$$

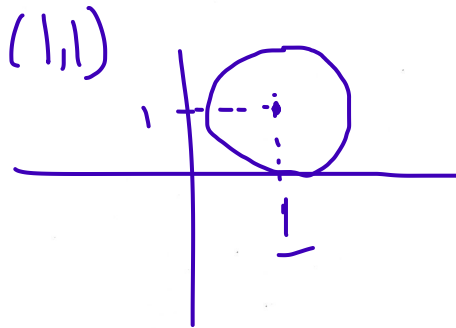
$$R = 10 \frac{\pi}{180}$$

$$D = R \frac{180}{\pi}$$

In a Cartesian coordinate plane, a circumference centered in  $C(1, 1)$  and tangent to the  $x$ -axis has equation:

$$(x-a)^2 + (y-b)^2 = R^2$$

$$(x-1)^2 + (y-1)^2 = 1^2$$



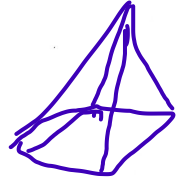
- ☐ A.  $x^2 + y^2 - 2x - 2y = 0$
- ☐ B.  $x^2 + y^2 - 2x - 2y = 1$
- ☐ C.  $x^2 + y^2 - 2x - 2y + 1 = 0$
- ☐ D.  $x^2 + y^2 - 2x + 2y = 0$
- ☐ E.  $x^2 + y^2 + 2x + 2y = 2$

$$x^2 - 2x + 1 + y^2 - 2y + 1 - 1 = 0$$

$$x^2 + y^2 - 2x - 2y + 1 = 0$$



We want to paint the surface of a solid. Which of the following requires the least amount of paint, all other things being equal (material, roughness, level of cleanliness, etc.)?



مکی کسرت

- !
- ☐ A. A pyramid whose faces are all equilateral triangles (tetrahedron) of side 1 meter.
  - ☒ B. A cube of side 1 meter.
  - ☐ C. A cylinder (right circular) of height 1 meter and base radius 1 meter.
  - ☐ D. A sphere of radius 1 meter.
  - ☐ E. A cone (right circular) of height 1 meter and base radius 1 meter.

$6a^2$

$4\pi r^2$

مکعب

استوانه

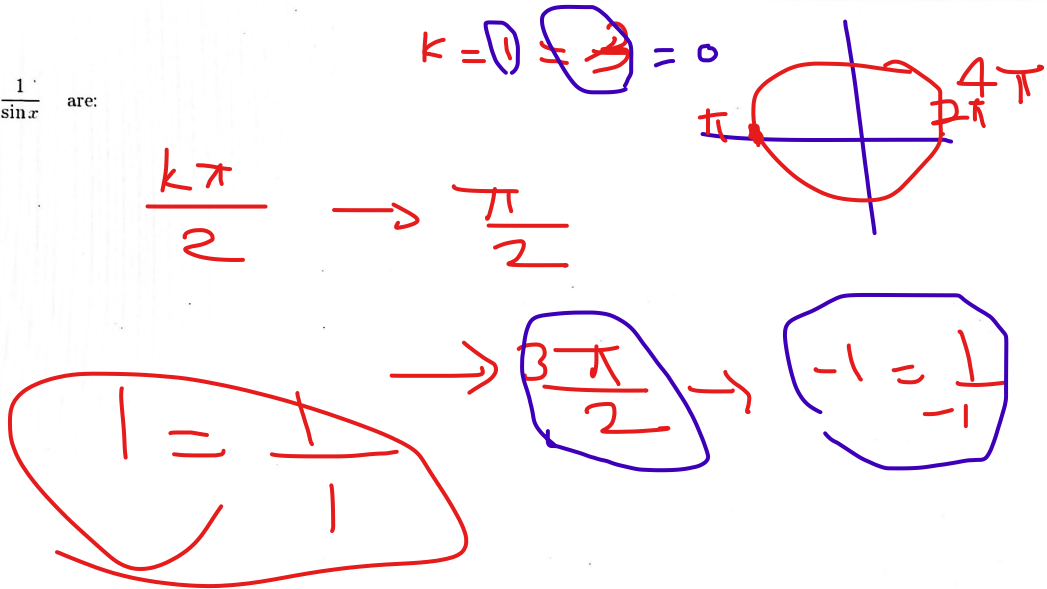
مخروط

$\pi r^2 + \pi r l$

$2\pi r^2 + 2\pi r h$

The solutions of the trigonometric equation  $\sin x = \frac{1}{\sin x}$  are:

- ☒ A.  $x = \frac{\pi}{2} + k\pi$ , for every integer value of  $k$
- ☐ B.  $x = \frac{\pi}{2} + 2k\pi$ , for every integer value of  $k$
- ☐ C. none of the other answers
- ☐ D.  $x = \frac{3\pi}{2} + 2k\pi$ , for every integer value of  $k$
- ☐ E.  $x = \frac{k\pi}{2}$ , for every integer value of  $k$



The expression  $\log_{10} \sqrt[3]{x^2+1} \cdot \log_{10} 1000$  is equivalent to:

Handwritten work showing the expression  $\log_{10} \sqrt[3]{x^2+1} \cdot \log_{10} 1000$  being simplified to  $\log_{10} (x^2+1)^{\frac{1}{3}} + 10^3$ .

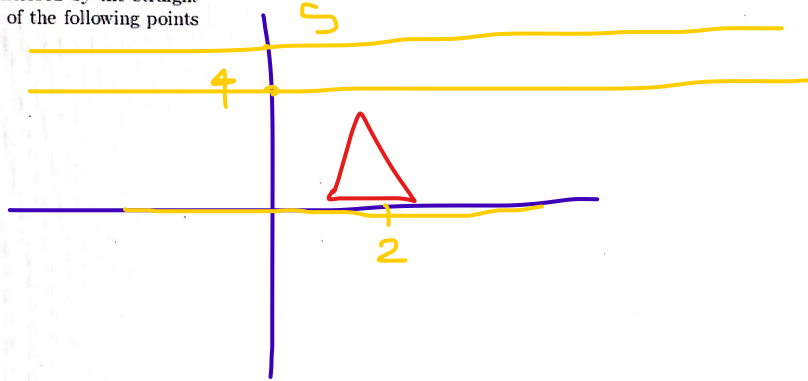
- ☐ A.  $\frac{1}{3} \log_{10} [1000(x^2+1)]$
- ☒ B.  $\log_{10} (x^2+1)$
- ☐ C.  $\log_{10} \sqrt[3]{x^2+1} + \log_{10} 1000$
- ☐ D.  $\log_{10} (1000 \sqrt[3]{x^2+1})$
- ☐ E.  $\log_{10} \frac{1000(x^2+1)}{3}$

Handwritten work showing the expression  $\log_{10} x^2+1$  being simplified to  $\log_{10} x^2+1$ .

$$x = 7 - y$$

In a Cartesian coordinate plane, consider the triangle enclosed by the straight lines  $r_1 : y = 0$ ,  $r_2 : y = 2x$ ,  $r_3 : y = -x + 7$ . Which of the following points lies inside this triangle?

- ☐ A.  $P = (4, 4)$
- ☐ B.  $P = (3, 5)$
- ☐ C.  $P = (3, 3)$
- ☐ D.  $P = (-3, 2)$
- ☐ E.  $P = (1, -3)$



The sum of the internal angles of an irregular hexagon:

- ☐ A. is equal to 360 degrees
- ☐ B. is equal to 6 right angles
- ☐ C. is equal to  $4\pi$  radians
- ☐ D. is equal to 5 straight angles
- ☐ E. cannot be found without further data

The Cambridge University Boat Club has a crew of six rowers, all equally skilled and working well together. Four of them must be chosen to represent Cambridge at The Boat Race against Oxford. How many ways are there to choose this delegation?

- ☐ A. 4
- ☐ B. 15
- ☐ C. 720
- ☐ D. 6
- ☐ E. 5

Let  $\gamma$  be a circumference and let  $P$  be a point in the plane that lies inside  $\gamma$  (but distinct from its center). Find the number of circumferences centered in  $P$  and tangent to  $\gamma$ .

- ☐ A. 2
- ☐ B. 4
- ☐ C. 0
- ☐ D. 1
- ☐ E. 3

Given a positive real number  $x$ , then  $\log(x^3) - \log(x^2)$  is equal to:

- ☐ A.  $\frac{\log(x^3)}{\log(x^2)}$
- ☐ B.  $\log(x^5)$
- ☐ C.  $\log(x)$
- ☐ D. 0
- ☐ E.  $\log(x^3 - x^2)$