English TOLC

Maths exercises with solutions

1402

آکادمی بین‌المللی دان

mailto:info@daanacademy.com

DAAN academy

Index

Introduction

This book was created for students who do not speak Italian, and due to the lack of content and resources to practise for the English TOLC-I exam. I decided to translate the exercises provided in Italian by CISIA and explain the solutions in English. I once was at your position and I think this would be very helpful.

Being able to solve all the exercises of this book demonstrates that you are prepared to do very well in the exam. Without further ado, keep studying and good luck.

Exercises

1) The prime factor decomposition of the number 30^{13} is:

a) $2^{15}\, 3^{12}\, 7^{13}$

b) $2^{13}\, 3^{13}\, 5^{13}$

c) 30^{13}

d) $6^{13}\, 5^{13}$

2) The following expression $\frac{\left(3^{20}+3^{20}+3^{20}\right)^{1/3}}{(3^3)^2}$ equals:

a) 3^2

b) 1

c) 3

d) 1/3

e) 1/9

3) In a cartesian plane, the circle of centre C with coordinates (1,1) and tangent to the X axis has the equation:

a) $x^2 + y^2 + 2x + 2y = 2$

b) $x^2 + y^2 - 2x + 2y = 0$

c) $x^2 + y^2 - 2x - 2y + 1 = 0$

d) $x^2 + y^2 - 2x - 2y = 0$

e) $x^2 + y^2 - 2x - 2y = 1$

4) A square of paper is folded into two equal parts to form two overlapping rectangles. If we know that the perimeter of the rectangle is 12 cm, what is the area of the original square?

a) 9 cm^2

b) 36 cm^2

c) 24 cm^2

d) 72 cm^2

e) 16 cm^2

5) The positive integer numbers *a, b, c, d* are all different from each other and less than 6. If their sum is 12, determine their product:

a) 60

b) 24

c) 50

d) 40

e) 30

6) Fixed in the plane, a system of orthogonal axes *O(x,y)* we consider the point $A = (1,0)$ and $B=(0,2)$. For which point *C is the triangle ABC* **not** a right triangle?

a) $C = (0, -1/2)$

b) $C = (-1, 0)$

c) $C = (1, 2)$

d) $C = (-4, 0)$

e) $C = (0, 0)$

7) A quantity of liquid which fills a sphere of radius *K* is poured into cylinders with a base diameter *K* and height *K*. What is the minimum number of cylinders needed to perform this operation?

a) 5

b) 6

c) 3

d) 9

e) 4

8) How many real numbers *x* are solutions of the following equation: $\tan(2x - 5\pi) = -10^4$

a) 1

b) 5

c) Infinite

d) None

e) 2

9) The cartesian equation $x^2 + y^2 + 4x = \gamma$ with y a positive real number, represents:

a) a circumference centred in $(-2, 0)$ with radius $\sqrt{y + 4}$

b) a circumference centred in (0, -4) with radius $\sqrt{y + 2}$

c) a circumference centred in $(0, 0)$ with radius $\sqrt{y}$

d) a circumference centred in $(-4, 0)$ with radius $\sqrt{y + 2}$

e) a circumference centred in (0, -2) with radius $\sqrt{y + 4}$

10) A sphere inscribed in a cube; the ratio of the volume of the sphere to that of the cube is:

a) $\frac{\pi}{4}$

b) $\frac{\pi}{6}$

c) $\frac{2\pi}{3}$

d) $\frac{4\pi}{3}$

e) $\frac{\pi}{2}$

11) The expression $\log(x^4 + 2x^2 + \sin^2 x + \cos^2 x)$ is the same as

a) $4\log(1 + x)$

b) $[\log(1 + x^2)]^2$

c) $2\log(1 + x^2)$

d) $\log(x^4 + 2x^2) + \log(\sin^2 x + \cos^2 x)$

e) $2\log(1 + x + \sin x + \cos x)$

12) All other conditions being equal (material, rugosity, cleanliness, etc.) less amount of paint is needed to paint:

a) A right circular cone with 1m height and a base radius of 1m

b) A sphere with a radius of 1m

c) A cube with 1m side length

d) A tetrahedron pyramid with all faces as equilateral triangles which sides are 1m long

e) A right circular cylinder with 1m radius and 1m height

13) The equation $\sqrt[3]{x^3 + 8} < 0$ is true

a) only if $x < -1$

b) for any real value of x

c) only if $x < -2$

d) only if $x < 0$

e) only if $x < 1$

14) The equation $x^4 + 3x^2 - 4 = 0$ *has:*

a) two positive solutions and none negative

b) no solution

c) one positive solution and one negative

d) two negative solutions and none positive

e) two positive and two negative solutions

15) Let a be a real number greater than 1. The logarithmic numeric expression $\log_a \sqrt{\frac{a^2 \sqrt{a}}{a^{\frac{5}{2}}}}$ equals:

a) -1

b) a

c) e

d) 0

e) 1

16) For $0 \leq x \leq \frac{\pi}{2}$, the equation $\sqrt{3}\sin^2 x + \sqrt{3}\cos^2 x - 2\sin x = 0$ has the solution:

a) $\frac{\pi}{3}$

b) $\frac{\pi}{6}$

c) $\frac{\pi}{4}$

d) 0

e) $\frac{\pi}{2}$

17) The equation $\sqrt{x^2} - x = 0$ is verified

a) only for $x = -1$

b) only for $x \geq 0$

c) only for $x = 0$

d) only for $x = 1$

e) $\forall x \in R$

18) In a cartesian system *O(x,y)* the distance between the point of coordinate (-4,2) and the line of equation $x = 2$ is:

a) -2

b) 2

c) -6

d) 6

e) 4

19) If x is a real number, the inequality $x^4 + 5 < 0$ *is satisfied for:*

a) all x

b) $x = -5$

c) no values of x

d) $x > -5$

e) $x < -\sqrt[4]{5}$

20) The inequality $\cos x + \sin x \geq \sqrt{2}$ is verified in the range $0 \leq x \leq 2\pi$ for:

a) all x

b) $x = -\frac{\pi}{4}$

c) at least one x so that $\frac{\pi}{2} < x < \pi$

d) $x = \frac{\pi}{4}$

e) no x

21) In a cartesian plane, the points different from (-1, 2) are points if:

a) $y \neq 2$

b) $xy \neq -2$

c) $x \neq -1$

d) $x \neq -1$ or $y \neq 2$

e) $x \neq -1$ and $y \neq 2$

22) Aldo, Bea, Carlo, Dario, Ebe, Franco go by train and find a compartment of six seats free. Considering that Aldo and Bea have to stay near the window, how many different ways can the six friends be arranged in the compartment?

a) 48

b) 4

c) 240

d) 8

e) 10

23) For which x is the following inequality verified? $\sqrt{x^2 - 1} > 2x$

a) $x \geq -1$

b) $x \leq -1$

c) $-1 < x < 1$

d) for no real x

e) $x \geq 1$

24) The expression $\log_{10} \sqrt[3]{x^2 + 1} \cdot \log_{10} 1000$ is equal to:

a) $\log_{10} \frac{1000(x^2+1)}{3}$

b) $\log_{10}(x^2 + 1)$

c) $\log_{10} \sqrt[3]{x^2 + 1} + \log_{10} 1000$

d) $\frac{1}{3} \log_{10}[1000(x^2 + 1)]$

e) $\log_{10}(1000 \sqrt[3]{x^2 + 1})$

25) Let Q be a square of side ℓ, C_1 a circle circumscribed around Q, and C_2 a circle inscribed in Q. The ratio between the area of C_1 and the area of C_2 is:

a) 4

b) 2

c) $\sqrt{2}$

d) Depends on the value of ℓ

e) $2\sqrt{2}$

26) The set $\{(x,y) \in \mathbb{R}^2 : \ x \neq 0, \ \frac{y}{x} > 2\}$ is composed of

a) one part of the plane delimited by a hyperbole

b) a circular crown

c) two vertically opposite angles

d) half a plane

e) two halves of plane

27) The Circolo Canottieri Santerno is formed by six oarsmen all equally good and close to each other. You have to send a representation of four athletes to the regional championship. How many ways can such a representation be formed?

a) 720

b) 5

c) 15

d) 4

e) 6

28) The equation $\mathbf{\cos^2 x - \cos x - 2 \geq 0}$ is verified by:

a) no real value of x

b) $x = \pi + 2k\pi$ for any k integer

c) $x = 2k\pi$ for any k integer

d) any real value of x

e) $x = 3k\pi$ for any k integer

29) A right triangle rotating around its legs, generates two cones. The ratio between the volumes of the two cones is equal to the ratio of:

a) the major leg and the hypotenuse

b) the legs

c) the legs squared

d) the minor leg and the hypotenuse

e) the legs cubed

30) Which of the following equalities is satisfied for all real numbers x and y?

a) $3^{x+y}3^{x-y} = 3^{x^2 - y^2}$

b) $3^{x+y}3^{x-y} = (3^x)^2$

c) $3^{x+y}3^{x-y} = 3^{x^2} - 3^{y^2}$

d) $3^{x+y}3^{x-y} = 3^{x^2}$

e) $3^{x+y}3^{x-y} = 3^x(3^y3^{-y})$

31) Lewis has two sons who are 15 and 11 years old respectively. In 18 years, Lewis' age will be equal to the sum of his sons' age. How old is Lewis now?

a) 30

b) There is insufficient data to answer

c) 52

d) 26

e) 44

32) Let A be the set of odd or prime positive integer numbers, it is true that:

a) $12 \in A$

b) $98 \in A$

c) $13 \notin A$

d) $2 \in A$

e) $3 \notin A$

33) The equation x+1>0 x+1<0

a) Admits infinite solutions

b) If $\mathbf{h > 0}$ is a solution, then $\mathbf{x = h +} \pi$ is a solution as well

c) Admits no solution

d) Admits only one solution

e) Admits exactly two solutions

34) If x is a real number, what is the solution set of the following inequality? $\frac{x+3}{x+1} \geq 2$

a) $-1 < x \leq 2$

b) $x \leq 1$

c) $x \geq 1$

d) $x < -1$

e) $-1 < x \leq 1$

35) In a country where every citizen is required to pay 25% of their income tax, one year the rate is lowered to 20%. However, a one-off tax of €1000 was introduced that year. It can be said that, in that year, with relation to this operation:

a) Citizens with an income superior to €25 000 had to pay an amount increased by a fifth of what they would have paid under the previous year rules
b) The tax burden remained the same for everyone
c) Only citizens with an income exceeding €10 000 were given an advantage
d) Citizens with an income exceeding €25 000 were given an advantage
e) Only citizens with an income less than €20 000 were given an advantage

36) In a parallelogram of perimeter **2p** we find that:

a) At least one diagonal has length equal to **p**
b) All diagonals have smaller length than **p**
c) All diagonals have greater length than **p**
d) The sum of the lengths of the diagonals is less than **p**
e) A diagonal is longer than **p**, the other smaller than **p**

37) Find x such that $\cos(x) + \sin(x) = 0$.

a) $x = \frac{\pi}{4}$

b) $x = 0$

c) $x = \pi$

d) $x = \frac{\pi}{2}$

e) $x = \frac{3\pi}{4}$

38) In a cartesian plane, which of the following points is inside the triangle enclosed by the three lines r_1: $y = 0$, r_2: $y = 2x$, r_3: $y = -x + 7$

a) $P = (3, 5)$

b) $P = (4, 4)$

c) $P = (1, -3)$

d) $P = (3, 3)$

e) $P = (-3, 2)$

39) An equilateral triangle is inscribed in a circumference; the ratio of the length of the circumference to the triangle perimeter is:

a) $\frac{4\pi}{3}$

b) $\frac{\pi}{3}$

c) $\frac{\sqrt{3}\pi}{2}$

d) $\frac{2\sqrt{3}\pi}{9}$

e) $\frac{2\pi}{\sqrt{3}}$

40) Consider the equation $\sin(x) = 2 - k$, with $0 \leq x \leq \pi$. This equation has at least one solution if and only if:

a) $k \geq 1$

b) $1 \leq k \leq 2$

c) $k \leq 2$

d) $-1 \leq k \leq 1$

e) $1 \leq k \leq 3$

41) Given a real number **x**, the following relation $\frac{2^x \cdot 2}{\sqrt{4^{x+1}}}$ is:

a) $\frac{1}{2^x}$

b) 0

c) $\frac{1}{2}$

d) 2

e) 1

42) Consider an equilateral triangle ABC whose side is 2 cm long. Let D, E, and F be the midpoints of AB, BC, and AC respectively. Find the area of the rhombus DECF.

a) $\frac{\sqrt{3}}{2}$ cm^2

b) 2 cm^2

c) $\sqrt{3}$ cm^2

d) $\sqrt{2}$ cm^2

e) $\frac{1}{\sqrt{3}}$ cm^2

43) Given two circles, one of centre (0, 0) and radius 1, and the other one of centre (2, 2) and radius 1, let $d(P_1, P_2)$ be the distance of a generic point P_1 in the first circle and a generic point P_2 in the second circle. The minimum **m** of $d(P_1, P_2)$ as P_1 and P_2 vary is:

a) $m = \sqrt{2}$

b) $m = \frac{\sqrt{2}}{2}$

c) $m = \sqrt{2} - 1$

d) inexistent

e) $m = 2(\sqrt{2} - 1)$

44) In Jeremiah's piggy bank there's only €1 and €2 coins. For a total of €60. Which of the following statements is true?

a) If the piggy bank has at least 30 coins, then most of them are 2€ coins
b) The number of €1 coins cannot be equal to the number of €2 coins
c) If the piggy bank has less than 40 coins, then most of them are €2 coins
d) The number of €1 coins is probably less than the number of coins of €2
e) The number of €1 coins is probably greater than the number of coins of €2

45) The equation $\mathbf{x^2 - 3|x| + 2 = 0}$ has:

a) four solutions
b) three solutions
c) two solutions
d) one solution
e) no solution

46) The equation $|x - 1| = 1 - |x|$ has:

a) Exactly two solutions

b) Exactly three solutions

c) Exactly four solutions

d) Infinite solutions

e) No solution

47) A rectangle is formed by two squares whose sides measure 2 cm and seven squares whose sides measure 1 cm . The rectangle perimeter is:

a) 22cm

b) 18cm

c) 24cm

d) 20cm

e) 16cm

48) The remainder of the division of the polynomial $x^5 - 3x^4 + 3$ by $x + 1$ is:

a) -1

b) 1

c) 3

d) 0

e) x – 1

49) Let **n** be a positive integer , if $\mathbf{x_n}$ **is** the solution of the equation

$$\frac{x+1}{1} + \frac{x+2}{2} + \cdots + \frac{x+n}{n} = n$$

Which of the following statements is true?

a) $\mathbf{x_n \neq 0}$ for every n

b) If $n_1 < n_2$ then $x_{n1} < x_{n2}$

c) If $n_1 < n_2$ then $x_{n1} > x_{n2}$

d) $x_n = x_n + 1$ for every n

e) $x_n = x_{n+1}$ for every n

50) Given two concentric spheres of radius 1 and **r** (with $r < 1$), which value should r assume for the volume of the external part of the smaller sphere to be half of the greater sphere volume.

a) $r = \frac{1}{\sqrt{3}}$

b) $r = \frac{1}{\sqrt[3]{3}}$

c) $r = \frac{1}{\sqrt[3]{2}}$

d) $r = \frac{1}{2}$

e) $r = \frac{1}{\sqrt{2}}$

51) The solution to the trigonometric equation $\sin x = \dfrac{1}{\sin x}$ is:

a) $x = \frac{\pi}{2} + k\pi$, for every integer value of k

b) None of the answers

c) $x = \frac{k\pi}{2}$, for every integer value of k

d) $x = \frac{3\pi}{2} + 2k\pi$, for every integer value of k

e) $x = \frac{\pi}{2} + 2k\pi$, for every integer value of k

52) On a cartesian plane, the equation of the axis of the segment (0,0) and (2,2) is:

a) $x - y = 2$

b) $x = 1$

c) $y = x$

d) $x + y = 2$

e) $y = 1$

53) Consider two points $A = (0, 0)$ and $B = (1, 1)$ on a cartesian plane. The equation of the axis of the segment AB is:

a) $y = \frac{1}{2} - x$

b) $y = 2 - x$

c) $y = 1 - \frac{x}{2}$

d) $y = 1 - x$

e) $y = \frac{1 - x}{2}$

54) An angle measures 2 radians, therefore

a) Its sine is positive

b) Its sine and it's cosine has the same sign

c) Its an acute angle

d) Its tangent does not exist

e) Its cosine is positive

55) Consider a circle with centre O and a 1 cm-long radius. Let P be a point outside this circle. The tangent lines from P to the circle meet the circumference in two points A and B. We know that the area of PAOB is equal to $\sqrt{3}$ cm^2. What is the distance between P and O?

a) 3 cm

b) 2 cm

c) 4cm

d) $\frac{\sqrt{3}}{2}$ cm

e) $\frac{3}{2}$ cm

56) From the semicircle with diameter $AB = 2$ cm and centre 0, the semicircle of diameter AO is removed. The figure obtained by then, is rotated around AB with a turn of 360 degrees. The volume of the solid obtained is:

a) $\frac{25}{3}\pi\ cm^3$

b) $\frac{7}{6}\pi\ cm^3$

c) $\frac{5}{6}\pi\ cm^3$

d) $4\pi\ cm^3$

e) $\frac{28}{3}\pi\ cm^3$

57) Which of the following statements is true for each pair of the polynomials $\mathbf{p(x)}$ and $\mathbf{q(x)}$ of degree 3 with real coefficient and where $\mathbf{p(x) \neq q(x)}$

a) $p(x) + q(x)$ of degree 6 and $p(x) \cdot q(x)$ of grade ≤ 6

b) $p(x) + q(x)$ of degree 3 and $p(x) \cdot q(x)$ of grade 3

c) $p(x) + q(x)$ of degree 6 and $p(x) \cdot q(x)$ of grade ≤ 9

d) $p(x) + q(x)$ of degree 3 and $p(x) \cdot q(x)$ of grade ≤ 6

e) $p(x) + q(x)$ of degree ≤ 3 and $p(x) \cdot q(x)$ of grade 6

58) Let $\mathbf{f(x) = x^2 - x^3}$, now $\mathbf{f(x - 2)}$ equals:

a) $x^2 - x^3 + 2$

b) $(3 - x)\,(x - 2)^2$

c) None of the answers

d) $x^2 - x^3 - 2$

e) $x^2 - 2 - x^3 + 2$

59) Let x be a positive real number and $f(x) = \log_{10}(x)$. Then $f(10 \cdot x^{-2})$ is equal to:

a) $\frac{1}{f(x)}$

b) $2 - 2f(x)$

c) $1 - 2f(x)$

d) $\frac{1}{2f(x)}$

e) $-2f(x)$

60) There is an equilateral triangle A of side a and an equilateral triangle B of side 2a, we find that the area of B is:

a) Greater than area of A but less than 2A

b) Quadruple of A area

c) Greater than quadruple of A area

d) Double of A area

e) Not deductible from A

61) The number $\left(\frac{81}{\sqrt{64}}\right)^{1/4}$ is equivalent to:

a) $\frac{3}{\sqrt{2}}$

b) $\frac{3}{2\sqrt{2}}$

c) $\frac{24}{8^{5/4}}$

d) $\frac{24}{64}$

e) $\frac{3}{2}$

62) Given any real number x, $\log(x^3) - \log(x^2)$ is equal to:

a) $\log(x^5)$

b) $\frac{\log x^3}{\log x^2}$

c) $\log(x)$

d) 0

e) $\log(x^3 - x^2)$

63) A team of workers has to pave a circular square, when they arrive, they discover that the square has a double diameter than expected. How much asphalt is needed compared to the budgeted one?

a) It can't be answered if the radius is unknown.
b) A quantity π^2 times higher than the expected.
c) Twice as expected.
d) The quadruple of the expected.
e) A quantity 2π times than the expected.

64) In a group of 100 people, 51 speak English and 36 French, of which 12 speak both English and french. How many people do not speak English nor french?

a) 49
b) 15
c) 29
d) 13
e) 25

65) Consider all real numbers a of the form: $a = \dfrac{3n + 3 + (-1)^n}{n + 1}$

Where n is a positive integer, how many of the numbers are greater than 2,99?

a) Infinite but not all
b) None
c) One
d) Two
e) All

66) In a cartesian plane, how many are the points verified by all three conditions: $(x + y)^2 = 1, \quad x^2 + y^2 = 1, \quad x + y \leq 0$

a) One
b) Two
c) Infinite
d) None
e) Four

67) The polynomial $12a^2 - 18b^2$ is divisible by

a) $\sqrt{6}(a - b)$

b) $\sqrt{2}a - \sqrt{3}b$

c) $12a + 18b$

d) $12a - 18b$

e) $a - b$

68) In a cartesian plane, the place of points that verify the equation: $\mathbf{(y - 2x^2)(y^2 - 4) = 0}$ is:

a) The set formed by the coordinate points (1, −2), (1, 2)

b) The set formed by the coordinate points (1, 2), (-1, 2)

c) The union of a parabola and two lines

d) The intersection of an hyperbole and two lines

e) The intersection of a parabola and two lines

69) In a plane, two equilateral triangles can be rotated and translated freely one in respect to each other. Given any position of the triangles, their overlapping position can never be:

a) A trapezium

b) An hexagon

c) A rectangle

d) An equilateral triangle

e) A right triangle

70) Let p be a positive odd number and q the following odd number. We find that:

a) $q^2 - p^2$ is divisible by 16 and may not be divisible by 32
b) $q^2 - p^2$ can be odd
c) $q^2 - p^2$ is divisible by 2 and may not be divisible by 4
d) $q^2 - p^2$ is divisible by 4 and may not be divisible by 8
e) $q^2 - p^2$ is divisible by 8 and may not be divisible by 16

71) In a cartesian plane, the equation $\mathbf{(x - 1)^2 - y^2 = 0}$ locate:

a) Two incident lines
b) A parabola
c) Two points
d) A circumference
e) Two parallel lines

72) Let x be a non–zero real number. The equation $x + \frac{1}{x} = k$ has one and only one solution if:

a) k = 1
b) k = 3
c) k = -3
d) k = -1

e) $k = 2$

73) Let x be a rational number satisfying the following property: $\mathbf{x \leq y}$ for all $\mathbf{y}$ rational, so that $\mathbf{y > \sqrt{2}}$, which statement is true?

a) x is the largest rational number minor than $\sqrt{2}$

b) x is the smallest rational number greater than $\sqrt{2}$

c) $x = y$

d) $x < \sqrt{2}$

e) $x = \sqrt{2}$

74) Consider an isosceles right triangle T. The sum of the cosines of the internal angles of T is:

a) 2

b) 1

c) $\sqrt{3}$

d) $1 + \sqrt{2}$

e) $\sqrt{2}$

75) The sum of the internal angles of a non-regular hexagon are:

a) Equal to five straight angles

b) Non deductible without more information

c) Equal to 4π radians

d) Equal to 360 degrees

e) Equal to 6 right angles

76) 30% of the students enrolled in a university course passed the final exam during the first session. 10% of the remaining students passed the exam during the second session. How many students still have to pass the exam after the first two sessions? Express the result as a percentage of the total number of students enrolled in the course.

a) 37%

b) 63%

c) 70%

d) 60%

e) 40%

77) In a cartesian plane, which of the following lines is parallel to the line that passes through the points (1, 0) and (0,1)?

a) $2x + 3y = 0$

b) $x = y - 1$

c) $x = 2$

d) $x + y = 3$

e) $y = 1$

78) A rational number between $\sqrt{5}$ and $\sqrt{8}$ is:

a) 2,52

b) 1,98

c) 3,01

d) $(\sqrt{5})(\sqrt{8})/2$

e) $(\sqrt{5} + \sqrt{8})/2$

79) Arrange the numbers in ascending order: $7, \sqrt{47}, \sqrt{3} + \sqrt{27}$

a) $\sqrt{3} + \sqrt{27} < \sqrt{47} < 7$

b) $\sqrt{47} < \sqrt{3} + \sqrt{27} < 7$

c) $7 < \sqrt{47} < \sqrt{3} + \sqrt{27}$

d) $7 < \sqrt{3} + \sqrt{27} < \sqrt{47}$

e) $\sqrt{47} < 7 < \sqrt{3} + \sqrt{27}$

80) In a cartesian plane consider the lines r_k of equation:

$\mathbf{y = kx + 2k + 1}$,where k is a real parameter. Which of these statements is true?

a) The lines r_k are intersecting two by two, but there is no common point between them all

b) For $k = 0$ we do not obtain a linear equation

c) All r_k lines go through the point $(1, -2)$

d) All r_k lines go through the point $(-2, 1)$

e) All the lines r_k are parallel to each other

81) The expression $\left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right)^2$ is also equal to:

a) $1 - \frac{\sqrt{3}}{2}$

b) $\frac{3}{2}$

c) $1 - \frac{\sqrt{2}}{2}$

d) $\frac{1}{2}$

e) 1

82) A winery produces the re-owned pirlot, a mixture of equal parts of white pinot and red merlot. From a bottle, the producer takes ⅓ and replaces it with an identical part of only pinot, then he takes ¼ of the new blend and replaces it with an identical part of only pinot again. What fraction of the final mixture is composed of pinot?

a) 5/7
b) 7/12
c) 5/12
d) 2/3
e) 3/4

83) What is the remainder of the division of $\mathbf{2x^3 - 3x + 2}$ by $\mathbf{x - 2}$?

a) 8
b) −1
c) 12
d) −8
e) −12

84) For which of the following values of x is $\mathbf{\sin\left(\frac{x}{3}\right) = \frac{sin(x)}{3}}$ verified?

a) $x = \frac{\pi}{3}$
b) $x = 3\pi$
c) $x = 2\pi$
d) $x = \frac{\pi}{6}$

e) $x = \frac{\pi}{2}$

85) Let **y** be a circle and **P** a point inside **y** different from its centre. How many circumferences of centre **P** are tangent to **y**?

a) 4

b) 0

c) 1

d) 2

e) 3

86) A motorcyclist, in his trip of 600 km, makes use of the spare wheel so that at the end, the three wheels were used for the same distance. How many km will each wheel make at the end of the journey?

a) 350 km

b) 400 km

c) 450 km

d) 500 km

e) 200 km

87) The equation $\mathbf{x(x - a) = 1}$ has two different solutions

a) Only if $\mathbf{a \geq 0}$

b) Only if $\mathbf{-1 < a < 1}$

c) For no real value of a

d) For all real value of a

e) Only if $-2 < a < 2$

88) In a cartesian plane, we are given the circle of equation: $\mathbf{\sqrt{3}x^2 + \sqrt{3}y^2 - 2x - 2y = 0}$. Its radius is:

a) $\sqrt{\frac{2}{3}}$

b) 3

c) $\sqrt{3}$

d) 1

e) 2

89) If **a** and **b** are real numbers so that $\mathbf{a^2 + b^2 = 0}$, we can certainly conclude that:

a) $a > b$

b) $ab < -1$

c) $a + b = 1$

d) $a + b = 0$

e) $ab > 0$

90) Given a regular hexagon of side L, the area of the rectangle that has two coincident sides with two parallel sides of the hexagon is equal to:

a) $2\sqrt{2}L$

b) $\sqrt{3}L^2$

c) That of the circle circumscribed to the hexagon

d) $2L^2$

e) That of the circle inscribed to the hexagon

Solutions

1) The correct answer is b.

Factoring a number into prime factors means expressing it as a product of **prime numbers** (numbers divisible by one or by themselves). The factorization of thirty is: $\mathbf{30 = 2 \cdot 3 \cdot 5}$

We also have the following rule of exponents: $(a \cdot b)^{\gamma} = a^{\gamma} \cdot b^{\gamma}$

$30^{13} = 2^{13} \cdot 3^{13} \cdot 5^{13}$

2) The correct answer is c.

Applying the rules of exponents we get:

$$\frac{\left(3^{20}+3^{20}+3^{20}\right)^{1/3}}{(3^3)^2} = \frac{\left(3 \cdot 3^{20}\right)^{1/3}}{(3^3)^2} = \frac{\left(3^{1+20}\right)^{1/3}}{3^{3 \cdot 2}} = \frac{3^{21 \cdot 1/3}}{3^6} = \frac{3^7}{3^6} = 3^{7-6} = 3$$

3) The correct answer is c.

A circle equation is $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) are the coordinates of the centre of the circle and r is the radius of the circle.

Therefore, we know that the equation will be $(x - 1)^2 + (y - 1)^2 = 1^2$ We know it because we are given the centre coordinates and, circle is tangent to the x axis, we know its radius is 1.

Although we reached the standard form of a circle equation, the options we are given are not in standard form, thus we need to transform the equation into something similar to the options. To do so, we can complete the perfect squares $(\mathbf{a} -- \mathbf{b})^2 = \mathbf{a}^2 - 2\mathbf{ab} + \mathbf{b}^2$

$(x - 1)^2 + (y - 1)^2 = 1^2 \Leftrightarrow x^2 - 2x - 1^2 + y^2 - 2y - 1^2 = 1 \Leftrightarrow$ **option** c.

4) The correct answer is e.

If we fold a square into two equal parts, the formed rectangle will have two sides with the original square length, and two sides with half the original length.

And since we know that the rectangle has a perimeter of 12cm, let ***x*** *be the length of any original square side*:

$12cm = 2x + 2(0.5x) \Leftrightarrow x = 12cm/3 \Leftrightarrow x = 4cm$

The area of a square is equal to the side length squared:
$(4cm)^2 = 16cm^2 \Leftrightarrow$ **option** e.

5) The correct answer is d.

Due to the restrictions of ***a, b, c, d*** being all positive numbers less than 6 we are left with the following numbers: {1,2,3,4,5}.

The sum of ***a, b, c, d*** is 12 and since ***a, b, c, d*** are all different between each other, the only possible operation is $1+2+4+5 = 12$ and the product of these numbers is $1 \cdot 2 \cdot 4 \cdot 5 = 40 =$ **option** d.

6) The correct answer is b.

After trying them, we can immediately discard options **c** and **e** since they are obviously forming a right triangle.

For the rest of the points, lets picture a **linear function**: $y = mx + b$

We know that if two lines intersect forming a right angle they have to be perpendicular lines. And to be perpendicular lines, their slopes (**m** in the linear function) need to be **negative reciprocals** between each other, $\mathbf{m_1 \cdot m_2 = -1}$ **.**

To calculate $\boldsymbol{m}$ (the slope), we have the following formula using two coordinate points: $m = \frac{y2 - y1}{x2 - x1}$ **;** for AB we easily calculate $\frac{y2 - y1}{x2 - x1} = m_{AB} \Leftrightarrow \frac{2 - 0}{0 - 1} = -2$. Now we need to find a $\boldsymbol{m}$ of the remaining options which is not equal to ½ (the negative reciprocal of -2).

That is the case with option b) C= (-1, 0). If we calculate the slope of CB: $\frac{2 - 0}{0-(-1)} = \mathbf{2}$, which is not the negative reciprocal of -2. Therefore is with the option b that we can form a not right triangle.

7) The correct answer is b.

We need to find how many cylinders with diameter and height $\boldsymbol{K}$ are needed to pour out all the water of the sphere with radius $\boldsymbol{K}$. The equations for the volumes of a sphere and cylinder are:

$V_s = \frac{4}{3}\pi K^3$ and $V_c = \pi(\frac{K}{2})^2 \cdot K$

Let ***x*** be the amount of cylinders needed:

$\frac{4}{3}\pi K^3 = x \cdot (\pi(\frac{K}{2})^2 \cdot K) \Leftrightarrow x = 5{,}\overline{33}$. Thus, we need 6 cylinders.

8) The correct answer is c.

The tangent function is a periodic function with period π. Therefore, $\boldsymbol{tan(2x - 5\pi) = -10^4} \Leftrightarrow \mathbf{tan(2x) = -10^4}$.

The tangent function has a range of (-∞, ∞), so we can be sure that $\mathbf{tan(2x) = -10^4}$ has a solution.

Due to the periodic property, we know that there are infinite solutions for the function. 2π ,2(2π), 2(3π)...

9) The correct answer is a.

A circle equation is $\boldsymbol{(x - h)^2 + (y - k)^2 = r^2}$ where $\boldsymbol{(h, k)}$ are the coordinates of the centre of the circle and ***r*** is the radius of the circle.

We can transform the given equation into the standard form above by completing its perfect squares:
$\mathbf{x^2 + y^2 + 4x = \gamma \Leftrightarrow (x^2 + 4x + 4) + y^2 = \gamma+4} \Leftrightarrow \boldsymbol{(x^2+2)^2 + y = y+4}$

Know we know that the circle has its centre in (-2,0) and its radius is $\sqrt{y + 4}$.

10) The correct answer is b.

The equations for the volumes of a sphere and a cube are:
$V_s = \frac{4}{3}\pi r^3$ and $V_c = l^3$ (with l being the side length of the cube)

Since the sphere is inscribed in the cube, we know that the side of the cube is two times the radius (diameter), therefore, to get the ratio requested:

$$\frac{\frac{4\pi r^3}{3}}{\frac{(2r)^3}{1}} = \frac{4\pi r^3}{3} \cdot \frac{1}{(2r)^3} = \frac{\pi}{6}$$

11) The correct answer is c.

Using the trigonometric identity $\mathbf{\sin^2 x + \cos^2 x = 1}$, we know that the expression is: $\mathbf{\log(x^4 + 2x^2 + 1)}$**.**

Similar to perfect squares, we have: $\mathbf{(x^4 + 2x^2 + 1) = (x^2 + 1)^2}$ therefore, we know that $\mathbf{\log(x^4 + 2x^2 + 1) = \log(x^2 + 1)^2}$.

Lastly, recalling the logarithm property: $\mathbf{\log_a b^\alpha = \alpha \log_a b}$ we have that: $\mathbf{\log(x^2 + 1)^2 = 2\log(x^2 + 1)}$

12) The correct answer is d.

After calculating all the areas the smallest one is the one from the tetrahedron.

13) The correct answer is c.

Just solve for $\sqrt[3]{x^3+8} < 0$ and you'll reach to $x < -2$.

14) The correct answer is c.

Let's transform this equation to one of second grade, let $\mathbf{t = x^2}$
$\mathbf{x^4 + 3x^2 - 4 = 0 \Leftrightarrow t^2 + 3t - 4 = 0}$

When solving this quadratic equation, we reach the following solutions: $\mathbf{t_1 = -4}$ and $\mathbf{t_2 = 1}$. Since $\mathbf{t = x^2}$, the solutions to the original equation are: **±√(-4)** and **±√1 .**

The square root of a negative number is undefined, thus, -4 is not a solution and we are left with two solutions: ± 1

15) The correct answer is d.

Using exponent rules, we get: $\log_a \sqrt{\frac{a^2\sqrt{a}}{a^{\frac{5}{2}}}} = \log_a \sqrt{\frac{a^2 a^{\frac{1}{2}}}{a^{\frac{5}{2}}}} = \log_a \sqrt{\frac{a^{\frac{5}{2}}}{a^{\frac{5}{2}}}} = \log_a 1$

$\mathbf{\log_a 1 = 0}$ since $\mathbf{a^0 = 1}$.

16) The correct answer is a.

Factoring and using the trigonometric identity $\mathbf{\sin^2 x + \cos^2 x = 1}$ we have:

$\sqrt{3}\sin^2 x + \sqrt{3}\cos^2 x - 2\sin x = 0 \quad \Leftrightarrow \quad \sqrt{3}(\sin^2 x + \cos^2 x) - 2\sin x = 0 \quad \Leftrightarrow \quad \sqrt{3} - 2\sin x = 0$

Solving the equation we get to $\sin x = \frac{\sqrt{3}}{2}$ and then $\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

17) The correct answer is b.

When solving the equation we get to $\sqrt{x^2} = |x|$. Therefore, we know that the equation is verified only for $x \geq 0$.

18) The correct answer is d.

If we graph we see that the distance between the point (-4,2) and the line **x=2** (a parallel line to the y-axis) is 6 units.

19) The correct answer is c.

When we solve the inequality we get to $\mathbf{x^4 < -5}$, and that is not possible since $\mathbf{x^4}$ is always a positive number.

20) The correct answer is d.

This inequality is solved by any x between 0 and $\frac{\pi}{2}$ (you can check it with the unit circle), the only answer in this range is d, with a value of $\mathbf{x = \frac{\pi}{4}}$.

21) The correct answer is d.

The point has to be different to (-1, 2), any point with $\mathbf{x \neq -1}$ or $\mathbf{y \neq 2}$ would satisfy this.

22) The correct answer is a.

Since Aldo and Bea have only two possible combinations between them (exchanging the window seats between them), the remaining people to combine are 4.

If we use the permutation formula (not combination because we are calculating the different ways they could be arranged, thus, the order matter) on the remaining people we have that:

$P_{(n,r)} = \frac{n!}{(n-r)!} \Leftrightarrow \quad 24 = \frac{4!}{(4-4)!}$ (the factorial of zero is one, **$0! = 1$**)

And since Aldo and Bea can exchange seats between them, we multiply 24 x 2 = 48. There are 48 possible rearrangements.

23) The correct answer is b.

Given the inequality we know the following things:

- Since $\mathbf{x^2-1}$ is square rooted, $\mathbf{2x < 0 \Leftrightarrow x < 0}$

- Since $\mathbf{x^2-1}$ is square rooted, $\mathbf{x^2 - 1 \geq 0}$ that gives us two solutions: $x_1 \geq \mathbf{1}$ or $\mathbf{x_2 \leq -1}$

Only x_2 agrees with the first statement therefore $\mathbf{x \leq -1}$ is the answer. Note that we changed the direction of $\geq$ to $\leq$ in x_2 , that is because when multiplying both sides of an inequality by a negative number, the direction of the inequality sign is reversed.

24) The correct answer is b.

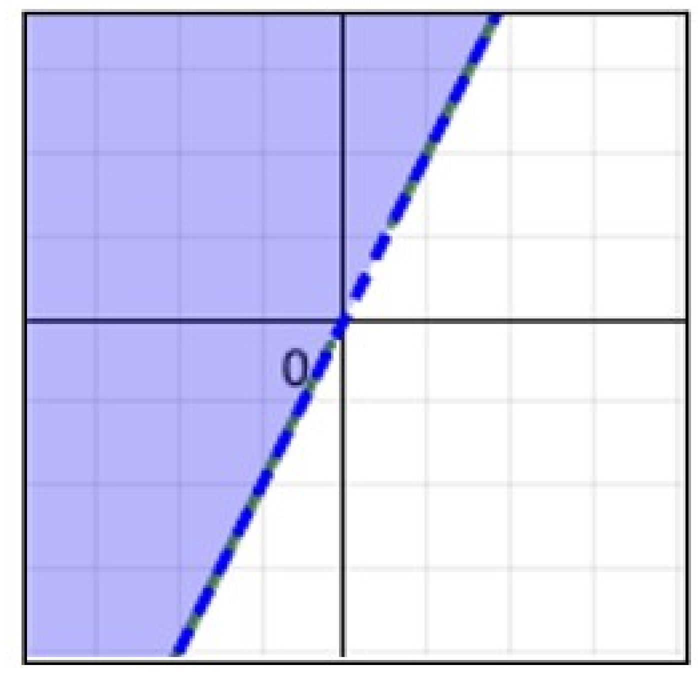
Since $\mathbf{\log_{10}1000 = 3}$ and the logarithmic property $\log_a b^\alpha = \alpha \log_a b$. We end with $\frac{1}{3}\log_{10}(x^2+1)\cdot 3 = \log_{10}(x^2+1)$

25) The correct answer is b.

The diameter of C_1 is the diagonal of Q ($\ell\sqrt{2}$), we can calculate it with the Pythagorean theorem. Also, the diameter of C_2 is ℓ. Then we can calculate the circles' areas, and when we calculate the ratio of C_1 between C_2 we discover that the result is 2.

26) The correct answer is d.

The equation can be written as: $\mathbf{y > 2x}$, its graph:



27) The correct answer is d.

To solve this we need to use the combination formula: $\mathbf{C_{(n,r)}} = \frac{n!}{r!(n-r)}$

where **n** is the amount of objects available and **r** is the amount of objects to choose.

28) The correct answer is b.

We need to find an angle such that $\mathbf{\cos^2 x - \cos x - 2 \geq 0}$ and that only happens when $\mathbf{\cos(x) = -1}$ and if we take a look at the unit circle, this is true when $x = \pi$, and due to cosine periodicity , every multiple of 2π we have the same cosine value, thus, c is the answer.

29) The correct answer is b.

Let **b** and **c** be the lengths of the triangle legs, with the formula of a cone's volume, we have the following ratio: $\frac{V_1}{V_2} = \frac{\pi b^2 c}{\pi c^2 b} = \frac{b}{c}$.

Note: the ⅓ part of the cone's volume formula gets cancelled out.

30) The correct answer is b.

Applying the exponent rule $\mathbf{a^x \cdot a^y = a^{x+y}} \Leftrightarrow \mathbf{3^{x+y}\, 3^{x-y} = 3^{x+x+y-y} = 3^{2x}}$.

And that's exactly what we get when we apply the other exponent property: $(a^x)^y = a^{xy} \Leftrightarrow \mathbf{(3^x)^2 = 3^{2x}}$

31) The correct answer is e.

If in 18 years Lewis will have the same age of his sons, we know that Lewis' age in 18 years: $15 + 18 + 11 + 18 = 62$. But we are asked Lewis' age **now** so, we can subtract 18 from 62 and we get to 44.

32) The correct answer is e.

We discard option **a)** and **b)** since they are even numbers which are not prime, so they are not included. We discard options **c)** and **e)** since they are odd numbers, thus, they should be included in the set. Finally, option d is true, since 2 is prime and it's included in the set.

33) The correct answer is d.

For sin(x) = -x , only 0 verifies the equation. You can check that with any other value from the unit circle, the equation will not be true.

34) The correct answer is e.

First we know that x ≠ -1 because the denominator is x + 1 and we cannot divide by 0. Then, subtracting 2 from both members we get:

$$0 = 2 - 2 \leq \frac{x+3}{x+1} - 2 = \frac{x+3-2(x+1)}{x+1} = \frac{x+3-2x-2}{x+1}$$

$$\Updownarrow$$

$$\frac{1-x}{x+1} \geq 0.$$

Because of the sign rule we know that the ratio of both quantities will only be positive if both have the same sign, therefore we have the following system of inequalities:

$$\begin{cases} 1 - x \geq 0 \\ 1 + x > 0 \end{cases} \qquad \begin{cases} 1 - x \leq 0 \\ 1 + x < 0. \end{cases}$$

The first one has solution $-1 < x \leq 1$; the second has no solution.

35) The correct answer is d.

We can solve the following equation to understand who got an advantage: $\mathbf{0.25x = 0.20x + 1000}$ where $\mathbf{x = 20\,000}$ we are basically saying that paying 25% of the income is the same as paying 20% plus €1000. So, whatever income higher than x will have an advantage.

36) The correct answer is b.

For this exercise I imagined a square (parallelogram) of 1 cm each side, thus, a perimeter of 2p where p=2. If we calculate any of the diagonals we find that, due to the special isosceles triangle their length is $\sqrt{2}$ which is smaller than 2 (p).

37) The correct answer is e.

For $\mathbf{cos(x) + sin(x) = 0}$ to be true there has to be an angle where its sine and cosine are the same value but one negative and the other one positive. Having a glance at the unit circle, we now that happens at the angle $\frac{3\pi}{4}$.

38) The correct answer is d.

For this exercise, you can graph the 3 lines and check which point is inside the triangle, you'll realise that the answer is d.

39) The correct answer is d.

Recall that the side of an equilateral triangle inscribed in a circle equals r $\sqrt{3}$, and the circumference equals $2\pi r$, therefore we can say that the ratio is:

$$\frac{2\pi r}{3r\sqrt{3}} \Leftrightarrow \frac{2\pi}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Leftrightarrow \frac{2\sqrt{3}\pi}{9}$$

40) The correct answer is b.

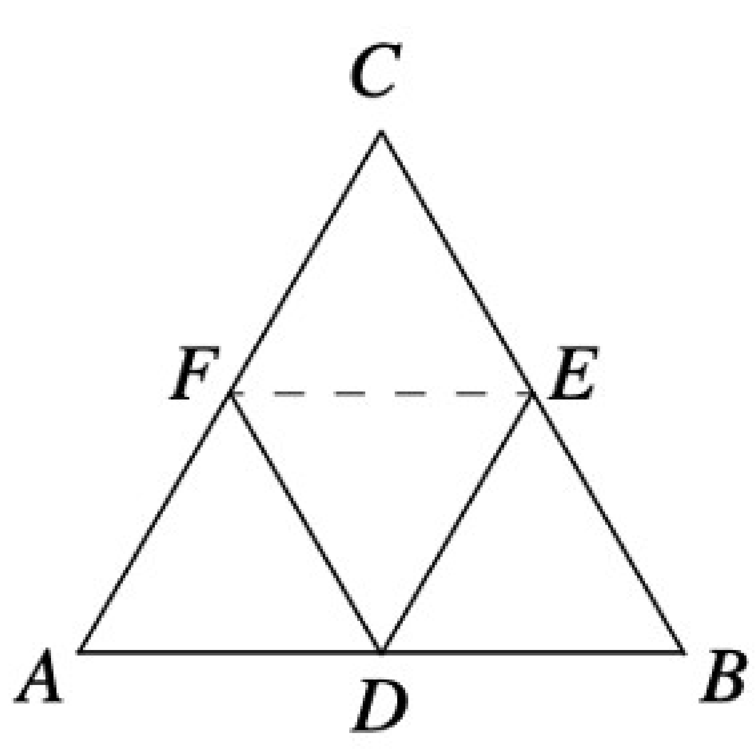
We have the constraint $0 \leq x \leq \pi$, and we know that in this range the sin(x) is between 0 and 1. Therefore, for $\mathbf{\sin(x) = 2 - k}$ to be true with this constraint, k has to be $1 \leq k \leq 2$.

41) The correct answer is e.

Applying exponent rules:

$$\frac{2^x \cdot 2}{\sqrt{4^{x+1}}} \Leftrightarrow \frac{2^{x+1}}{\sqrt{(2^2)^{x+1}}} \Leftrightarrow \frac{2^{x+1}}{(2^{2x+2})^{1/2}} \Leftrightarrow \frac{2^{x+1}}{2^{x+1}} \Leftrightarrow 1$$

42) The correct answer is a.



If we draw the instructions we get the graph on the right, therefore, we know they are asking for half of the area of the triangle. We can draw a line from C to D so that we have two special triangles with angles 30, 60 and 90, therefore, we end with two triangles with sides of 2cm, 1cm and $\sqrt{3}$ cm. After calculating the triangle's area and a half, we reach to option a.

43) The correct answer is e.

If we draw the points (0,0) and (2,2) we can calculate the distance between them by drawing isosceles triangles. Since these triangles are isosceles, thus, with 45 degree angles, we can use the special triangle rule to calculate the hypotenuse which is $\sqrt{2}$.
Since we have two triangles between the points, and we are calculating the minimum distance between the circumferences, we can say that the distance is two times the hypotenuse minus the radius of the circumference: $2(\sqrt{2} - 1)$.

44) The correct answer is c.

We can discard option a because, if we have 20 coins of 2€ and 20 coins of 1€ we'll have 60€. Because of the same reason we can discard options b, d and e.
Therefore, option c is correct.

45) The correct answer is a.

Since we have **3|x|,** we know 3|x| can take either positive or negative values. Therefore,we have two options:

i) $\begin{cases} x \geq 0 \\ x^2 - 3x + 2 = 0 \end{cases}$ ii) $\begin{cases} x < 0 \\ x^2 + 3x + 2 = 0 \end{cases}$

Solving both quadratic functions we get that:

i) $\begin{cases} x \geq 0 \\ x = 2 \lor x = 1 \end{cases}$ ii) $\begin{cases} x < 0 \\ x = -1 \lor x = -2 \end{cases}$

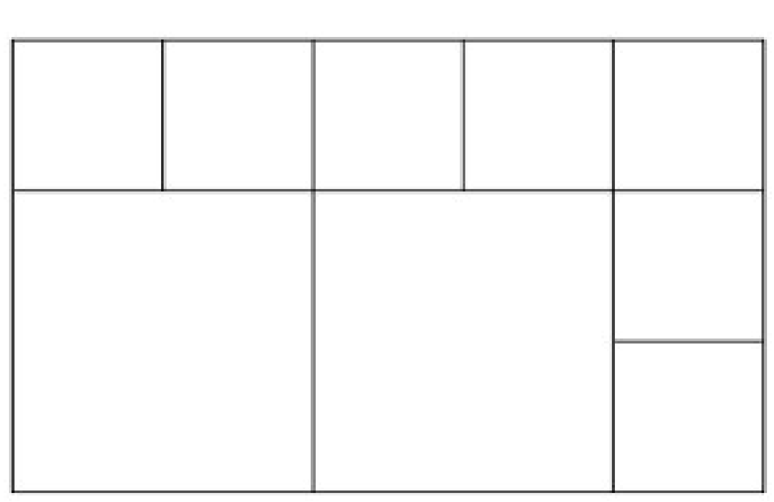
With four different solutions, the answer is a.

46) The correct answer is d.

If $\mathbf{x} \in \mathbf{[0, 1]}$ we find that $|x-1| = 1 - x$ and $|x| = x$, therefore we can write the equation as it follows: $\mathbf{1 - x = 1 - x}$ which is an equality verified by infinite x values in the interval [0, 1]

47) The correct answer is e.

The triangle will have two sides with 5 cm and two with 3 cm as it follows:



48) The correct answer is a.

When doing the polynomial division, you will see that the remainder is -1

49) The correct answer is e.

I have no idea why, but if you know, please tell me. (arithmetic?)

50) The correct answer is c.

We know that half the volume of the sphere with radius 1 should equal to the external part of the smaller sphere. We can get this external part volume by subtracting the smaller sphere volume to the greatest sphere volume, written as an equation:

$$\frac{2}{3} \cdot \pi \cdot 1^3 = (\frac{4}{3} \cdot \pi \cdot 1^3) - (\frac{4}{3} \cdot \pi \cdot x^3)$$

When we solve this equation we reach to the solution $\frac{1}{\sqrt[3]{2}}$

51) The correct answer is a.

This equation is solved only when sin(x) = 1 or sin(x) = -1, and by looking at the unit circle, we see that only happens at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

52) The correct answer is a.

When drawing the axis of the segment provided, we see that it passes through the y axis at (0, 2) and that the slope is -1, since it is the negative reciprocal of the original segment. Thus we have the linear equation $y = -x + 2$

53) The correct answer is d.

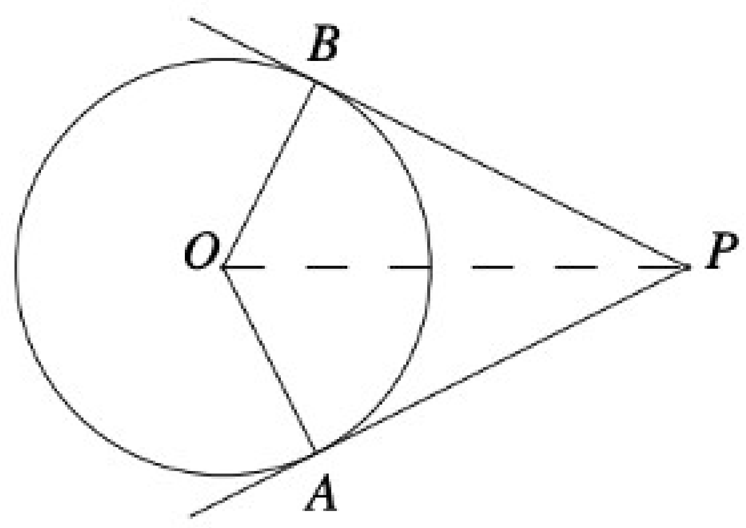
When drawing the axis of the segment provided, we see that it passes through the y axis at (0, 1) and that the slope is -1, since it is the negative reciprocal of the original segment. Thus we have the linear equation $y = -x + 1$

54) The correct answer is a.

We need to picture the unit circle; we know that half the circle is π radians, so if this angle measures 2 radians, we know that it will be in the second quadrant, therefore its sine is positive.

55) The correct answer is b.

If we draw the circle with centre O , the point P and its tangents touching the circle in A and B we get the following:



We see that if we draw the segment OP we get two triangles. We also know that the area of PAOB is $\sqrt{3}\ cm^2$, so we can calculate BP by using the area formula:

$\mathbf{OB \cdot BP = AREA \Leftrightarrow 1 \cdot BP = \sqrt{3} \Leftrightarrow BP = \sqrt{3}}$, and now, using the pythagorean theorem, we can calculate the hypotenuse which would be the segment OP: $\mathbf{1^2 + (\sqrt{3})^2 = hypotenuse^2 \Leftrightarrow \sqrt{4} = hypotenuse}$

56) The correct answer is b.

When rotating a semicircle by 360 degrees around its diameter, we get a sphere. Thus, to calculate the volume of the solid obtained, we need to calculate the volume of the sphere with diameter AB, and subtract to it the volume of the sphere removed, the equation:

$$\mathbf{V_s = (\frac{4}{3} \cdot \pi \cdot 1^3) - (\frac{4}{3} \cdot \pi \cdot (½)^3) = \frac{4\pi}{3} - \frac{4\pi}{24} = \frac{32\pi}{24} - \frac{4\pi}{24} = \frac{28\pi}{24} = \frac{7\pi}{6}}$$

57) The correct answer is e.

The sum of the polynomials can result in a polynomial of degree 3 or not depending on the signs of the x^3 term. (if subtracted and eliminated there is no more degree 3) and it's multiplication will always be of grade 6, since $x^3 \cdot x^3 = x^6$.

58) The correct answer is b.

f(x – 2) would be $\mathbf{(x-2)^2 - (x-2)^3}$**,** if we factorise it with $\mathbf{(x-2)^2}$ we get $\mathbf{(x-2)^2\,[1-(x-2)] = (x-2)^2\,(1-x+2) = (x-2)^2(3-x)}$**.**

59) The correct answer is c.

Due to the property of exponents, we know that $\mathbf{x^{-n} = \frac{1}{x^n}}$**.** Therefore, we have that $f(10\cdot x^{-2}) = \log_{10}(\frac{10}{x^2})$. And due to the logarithmic properties $\mathbf{\log_b(x/y) = \log_b x - \log_b y}$ and $\mathbf{\log_b x^n = n \log_b x}$ we can assume: $\mathbf{\log_{10}(\frac{10}{x^2}) = \log_{10}10 - \log_{10}x^2 = 1 - 2\log_{10}x =}$ $1 - 2f(x)$

60) The correct answer is b.

To calculate the area of an equilateral triangle we can draw a line from one of its vertex to the bottom to divide it into two right triangles, then because of the special triangle (30, 60, 90) rule, we know that this new drawn segment is $\frac{\sqrt{3}a}{2}$ in A and $\sqrt{3}$a in B. Now that we have the height we can calculate the area of both triangles by multiplying the area and the base of the special triangles. For A we get $\frac{\sqrt{3}a}{2} \cdot \frac{a}{2} = \frac{\sqrt{3}a^2}{4}$, for B: $\sqrt{3}a \cdot \mathrm{a} = \sqrt{3}a^2$.

Therefore, the area of B is 4 times the area of A.

61) The correct answer is c.

To get to option c, we do the following steps:

$$\left(\frac{81}{\sqrt{64}}\right)^{1/4} \Leftrightarrow \left(\frac{3}{8^{1/4}}\right) \Leftrightarrow \left(\frac{3 \cdot 8}{8^{1/4} \cdot 8}\right) \Leftrightarrow \left(\frac{24}{8^{5/4}}\right)$$

62) The correct answer is c.

We can use the logarithm rules: $\log(x^3) - \log(x^2) = 3\log(x) - 2\log(x) = \log(x)$

63) The correct answer is d.

Since, to get the area of a circle we use the radius, which is half the diameter, and in the area equation the radius is squared. We know that the final quantity is quadruple of the expected. Let's imagine a scenario where the original diameter is 2 and when doubled, 4:

$\frac{4}{3} \cdot \pi \cdot 1^2 = \frac{4\pi}{3}$, and with diameter doubled: $\frac{4}{3} \cdot \pi \cdot 2^2 = \frac{16\pi}{3}$

64) The correct answer is e.

We know that 12 of those who speak English, also speak French, and likewise. So we can calculate 100 – (51 – 12) – (36 -12) - 12 = 25

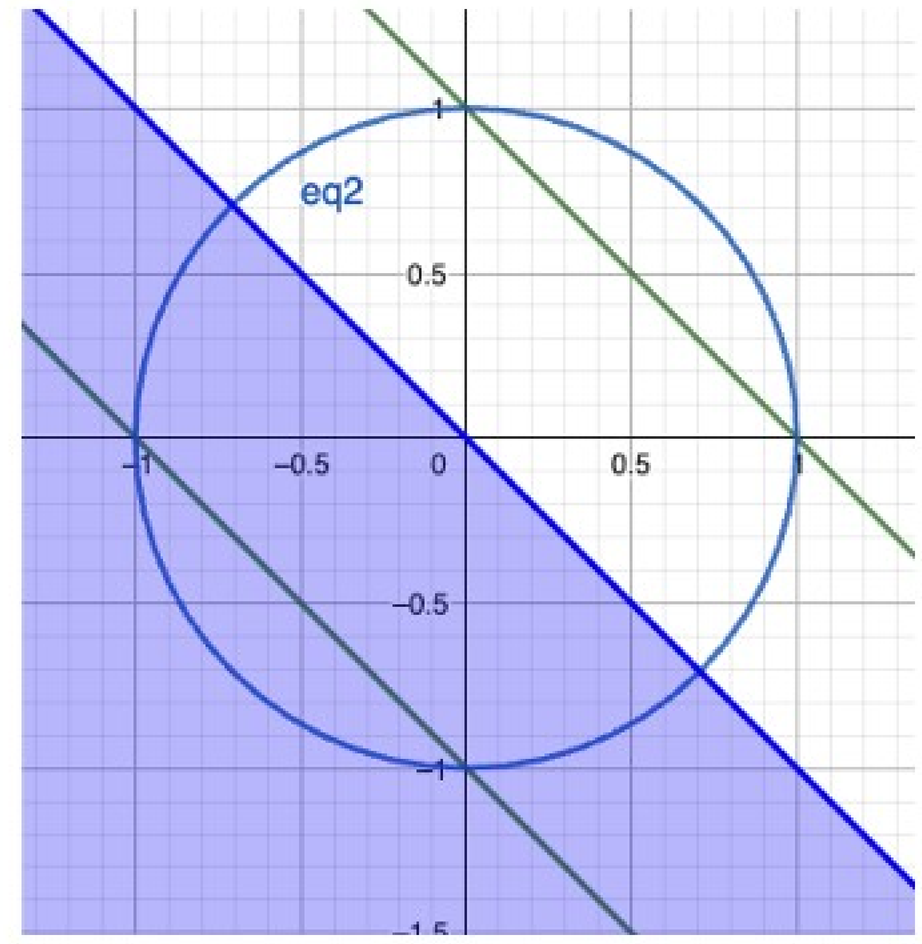
65) The correct answer is a.

We can write the number the following way $a = \frac{3(n+1)+(-1)^n}{n+1} = 3 + \frac{(-1)^n}{n+1}$.

Only positive and even **n**s will be greater than 2,99.

66) The correct answer is b.

The first equation $(x + y)^2$ are two parallel lines. If we square root both sides, we get that $x + y = \pm 1$, so two lines, depending on the sign of 1. The second one is a circumference with centre (0, 0) and radius 1, and the last one is half a plane, so if we draw all the equations we see that only 2 points intersect with all 3 equations.



67) The correct answer is b.

A polynomial is divisible by one of its factors.

$12a^2 - 18b^2 = 6(2a^2 - 3b^2) = 6(\sqrt{2}a + \sqrt{3}b)(\sqrt{2}a - \sqrt{3}b)$

Therefore, we know answer b is correct.

68) The correct answer is c.

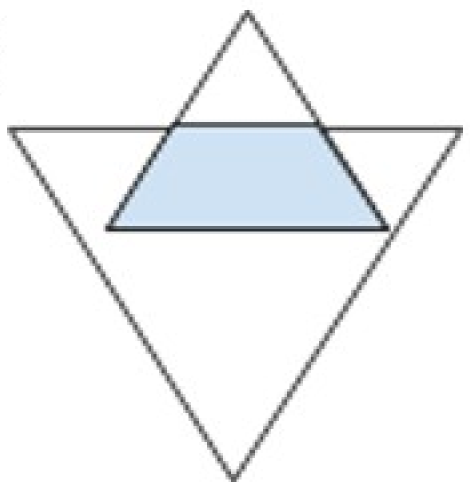
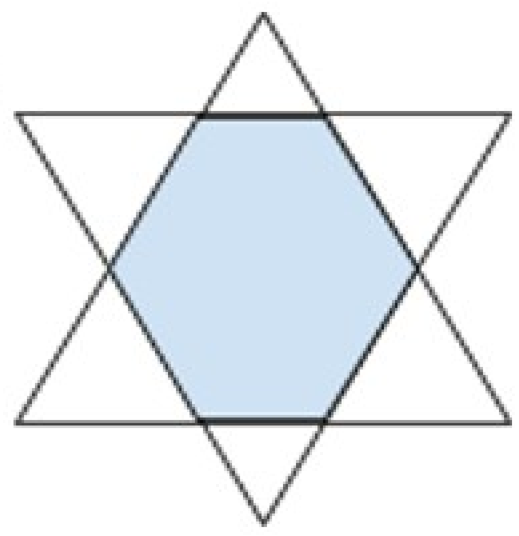
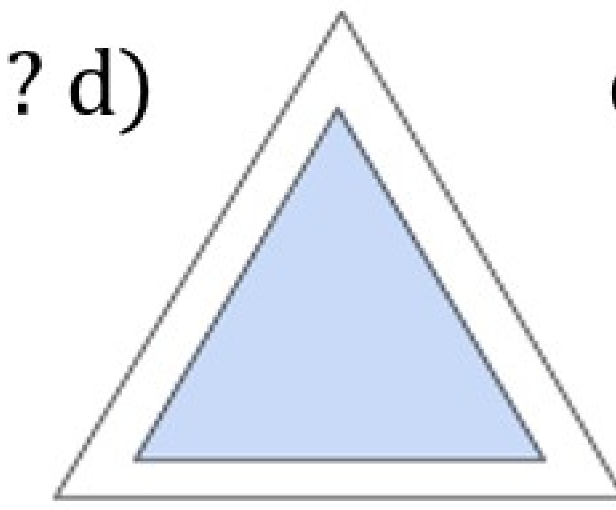
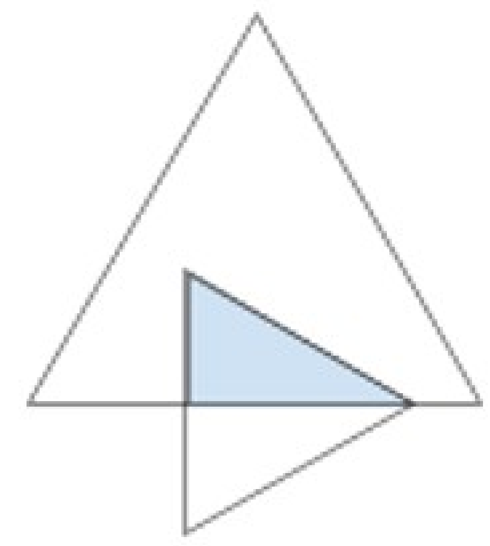
Dividing the equation by one of its products we get the following 2:

$y - 2x^2 = 0$ and $y^2 - 4 = 0$.

The first one is a parabola $y = 2x^2$, and the second one are two parallel lines $y = 2$ and $y= -2$.

69) The correct answer is c.

Let's try to get to each figure:

a)  b)  c) ? d)  e) 

The only figure not possible to draw was the rectangle.

70) The correct answer is e.

If p is a positive odd number, then it exists a natural positive number **h** so that: $\mathbf{p = 2h - 1}$ and **q** being the following odd number we also have that: $\mathbf{q = 2h + 1}$, we can calculate $\mathbf{q^2 - p^2}$ as it follows:

$$q^2 - p^2 = (2h + 1)^2 - (2h - 1)^2 = 4h^2 + 4h + 1 - (4h^2 - 4h + 1) = 8h$$

Therefore $\mathbf{q^2 - p^2}$ is divisible by 8 but if h is odd, not by 16.

71) The correct answer is a.

$$(x - 1)^2 - y^2 = 0 \Leftrightarrow (x - 1)^2 = y^2 \Leftrightarrow y = \pm (x - 1)$$

72) The correct answer is e.

If $x = 1$ we get $k = 2$. That's the only solution with an integer number apart from k= -2 with $x = -1$, which is not an option, therefore, e is the answer.

73) The correct answer is d.

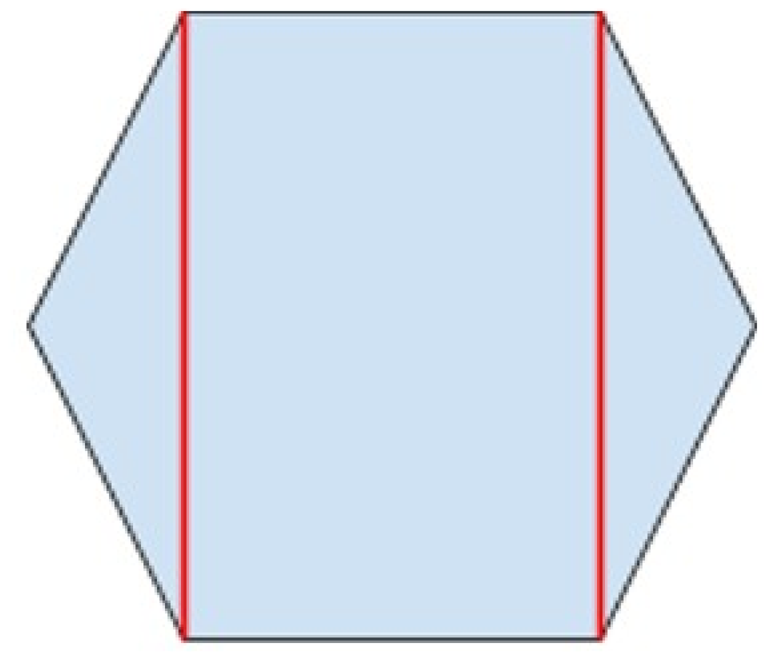
It cannot be $\mathbf{x > \sqrt{2}}$ because in this case, there would be a rational number **y** so that $\mathbf{\sqrt{2} < y < x}$ which does not comply with the hypothesis. Therefore, option b and c are false. It cannot be $x = \sqrt{2}$ because $\sqrt{2}$ is irrational. Therefore, option e is false. It cannot be option a, because there's a rational number x_1 so that $x < x_1 < \sqrt{2}$. So option d is correct.

74) The correct answer is e.

We know that a right isosceles triangle has one angle of 90 degrees and two angles of 45 degrees. The cosine of an angle of 90 degrees is zero. The cosine of an angle of 45 degrees, by looking at the unit circle we see that is $\frac{\sqrt{2}}{2}$, so $0 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

75) The correct answer is c.

We can calculate the sum of the internal angles of an hexagon by drawing to lines to form a rectangle in the middle of it:



Now, we can sum four 90 degrees angles inside the rectangle, and 180 degrees for each triangle to get 720 degrees = 4π

76) The correct answer is b.

At first we had 30% of students that passed the final exam. At the second session, 10% of the remaining students (70%) passed the exam. So, 10% of 70% = 7% . Therefore the students that still have to pass the exam after the two sessions is 100% - 30% - 7% = 63%

77) The correct answer is d.

Since the line passes through the points (1, 0) and (0, 1), we know that its equation is going to be $\mathbf{y = -x + 1}$, therefore a parallel line to this equation needs to have an identical slope (-x) that is the case with option d: $\mathbf{x + y = 3 \Leftrightarrow y = -x + 3}$

78) The correct answer is a.

We can discard options d and e since those options are not rational numbers, we also discard option c because: $\sqrt{8} < \sqrt{9} = 3 < 3{,}01$ And option b is discarded because $1{,}98 < 2 = \sqrt{4} < \sqrt{5}$. Therefore, option a is correct.

79) The correct answer is b.

To reach the answer we can square all the numbers:
$\mathbf{7^2, (\sqrt{47})^2, (\sqrt{3} + \sqrt{27})^2}$ which gives us the following numbers, easier to arrange $7^2 = 49$, $(\sqrt{47})^2 = 47$, and for $(\sqrt{3} + \sqrt{27})^2$:
$(\sqrt{3} + \sqrt{27})^2 = (\sqrt{3} + \sqrt{3 \cdot 9})^2 = (\sqrt{3} + 3\sqrt{3})^2 = (4\sqrt{3})^2 = 48.$

80) The correct answer is d.

Given a point P(x0, y0) in the cartesian plane, the equation for all the lines with centre point P has the equation $\mathbf{y - y0 = k(x - x0)}$.
If we replace this equation with the point (-2, 1) we get :
$\mathbf{y - 1 = k(x + 2) \Leftrightarrow y = kx + 2k + 1.}$

81) The correct answer is d.

We have a perfect square, therefore:

$$\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)^2=\sin^2\frac{\pi}{12}+\cos^2\frac{\pi}{12}-2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$$

Using the following trigonometry properties: $\sin^2\alpha+\cos^2\alpha=1$ and $\sin 2\alpha=2\sin\alpha\cos\alpha$ we obtain $\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)^2=1-\sin\frac{\pi}{6}$

82) The correct answer is e.

First we have a bottle with a mixture composed of ½ of each wine type.

a) When the producer replaces ⅓ of this mixture with only pinot, we get a bottle that has ⅔ of mixture with equal parts for each wine type, meaning it has ⅓ of pinot and ⅓ of merlot, and also another ⅓ part of pinot, just added by the producer.

b) Then the producer takes ¼ of the new blend and replaces it again with only pinot, so we are left with ¾ of the bottle with ⅔ of pinot and ⅓ of merlot and ¼ of only pinot.

$$\frac{3}{4}\cdot\frac{2}{3}+\frac{1}{4}=\frac{3}{4}$$

83) The correct answer is d.

When dividing the polynomials we get 12 as the remainder.
Also the remainder theorem affirms that the remainder of the division of a polynomial by a binomial of first degree is obtained by replacing x in the polynomial for the constant of the binomial with opposite sign: $R(2) = 2(2)^3 - 3(2) + 2 = 16 - 6 + 2 = 12$

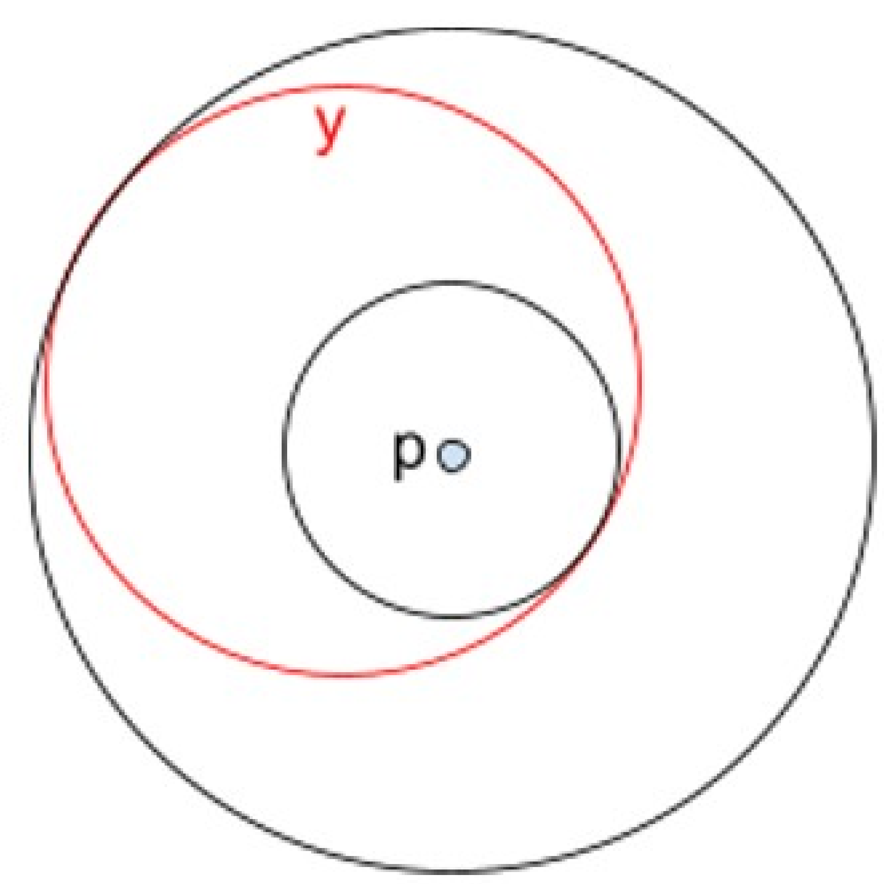
84) The correct answer is b.

Let's replace the equation with x = 3π:

$$\sin\left(\frac{3\pi}{3}\right) = \frac{\sin(3\pi)}{3} \Leftrightarrow \sin(\pi) = \frac{0}{3} \Leftrightarrow 0 = 0$$

85) The correct answer is d.

Only two circles can be tangent to y, as it follows:



86) The correct answer is b.

Let A, B, and C be the three wheels; If the motorcyclist does 200 km with A and B, then another 200 km with A and C, and the final 200 km with B and C. By the end each wheel would have been used for 400 km.

87) The correct answer is d.

We can turn it into a quadratic one: $\mathbf{x(x - a) = 1 \Leftrightarrow x^2 - xa - 1 = 0}$
Remember that a quadratic formula will only have 2 solutions if the discriminant is greater than 0. The discriminant in this case would be: $\mathbf{a^2 - 4 \cdot 1 \cdot (-1) = a^2 + 4}$, since any real value of a makes this equation greater than 0 , option d is correct.

88) The correct answer is a.

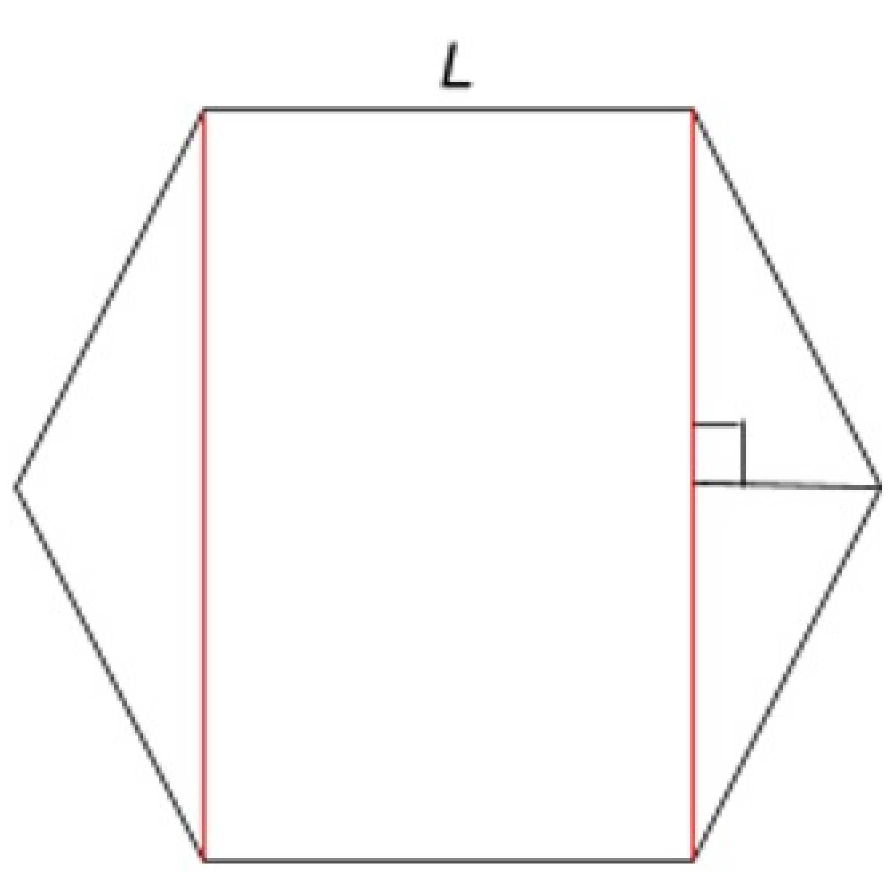
To form the circle equation we can use the following property of perfect squares: $\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$, completing the perfect squares: $\mathbf{\sqrt{3}x^2 - 2x + ⅓ + \sqrt{3}y^2 - 2y + ⅓ = ⅔}$ and since the radius is squared in a circle equation we know that the radius is $\sqrt{\frac{2}{3}}$

89) The correct answer is d.

The equation $\mathbf{a^2 + b^2 = 0}$ is only verified with real numbers if $\mathbf{a = 0}$ and $\mathbf{b = 0}$.

90) The correct answer is b.

When we draw this rectangle we see that there are two triangles on the sides, by tracing the height from the vertex of one of the triangles to the base we get a 30, 60, 90 special triangle with one side L, we now can calculate the side of the triangle which forms the side of the rectangle to then calculate the area:



Now we can calculate the area by calculating the side, which will be $2 \cdot \sqrt{3} \cdot \frac{L}{2} \cdot \mathrm{L} = \sqrt{3}\mathrm{L}^2$

This book was created for students who do not speak Italian, and due to the lack of content and resources to practise for the English TOLC-I exam. I decided to translate the exercises provided in Italian by CISIA and explain the solutions in English. I once was at your position and I think this would be very helpful.
Being able to solve all the exercises of this book demonstrates that you are prepared to do very well in the maths section of the exam. Without further ado, keep studying and good luck.

Translated by Theo Radicella
CISIA exercises translated to English
10th April, 2023
theoradicella@gmail.com